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DETERMINING THE TRANSFER FUNCTION OF A DYNAMIC SYSTEM WITH CONSTANT PARAMETERS

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ABSTRACT: It is shown how the method for determining the transfer function of a dynamic system with constant parameters developed by A. N. Sklyarevich (Sklyarevich, A. N. Operatornyye metody v statisticheskoy dinamike avtomaticheskikh sistem. "Nauka", M., 1965) can be extended to the case when the poles of the transfer function of the system and the Laplace transform of the correlation function of the input signal do not coincide and are of order higher than one. A procedure for determining the cross-correlation function of the input and output signals is presented. This cross-correlation function is used as a basis for deriving a system of algebraic equations for determining the coefficients of the frequency response function and a recurrence formula for their calculation is presented. It is stressed that the results obtained here can be utilized in designing a special-purpose computer for calculating the transfer functions. Two examples illustrate the calculation procedure. Orig. art. has: 26 formulas. English Translation: 11 pages.

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DETERMINING THE TRANSFER FUNCTION OF A DYNAMIC SYSTEM WITH CONSTANT PARAMETERS

B. B. Sodell'

This article examines a method of finding the pulsed transfer function of a linear dynamic system with constant parameters; the method is based on a comparison of the cross-correlation function of the input and output signals of the system with the correlation function of the input signal. Formulas are derived for the case when the poles of the transform of the input signal correlation function and the transfer function of the system do not coincide and have an order higher than one.

In an investigation of the quality and accuracy of operation of control systems during their normal exploitation the problem of determining the pulsed transfer function g(t) in a time region or of the transfer function of the system W(s) in the region of a complex variable can arise.

To find the pulsed transfer function g(t) for stationary systems with stationary random actions on the input, different methods of solving the integral equation

$$K_{yx}(\tau)=\int_0^{\infty} g(u)\,K_x(\tau-u)\,du \qquad (1)$$

relative to the function g(t) are used [1].

Let us briefly examine some of them.

1. Calculation of the pulsed transfer function g(t) by solving the approximating system of linear algebraic equations

$$K_{yx}(\tau) \approx \sum_{n=0}^{N} g(nT) K_x(\tau - nT) T. \qquad (2)$$

Equation (2) is solved by any of the known methods. Application of the Gauss-Seidel iteration methods makes it possible to use special-purpose computers (synthesizors).

2. Solution of equation (1) with a finite upper limit by the selection method. Selecting a value of g(t), we calculate the integral in the right part of the formula until the obtained value of the function coincides with the cross-correlation function $K_{yx}(\tau)$ determined from experimental data. "Control filters" are used to solve the problem by the selection method.

3. Determination of the frequency characteristics of the system. The basis of the frequency method is the reduction of integral equation (1) to an algebraic equation with the aid of a Fourier transform. As a result we determine the frequency characteristics of the system in the form

$$W(j\omega) = \frac{S_{yx}(j\omega)}{S_x(\omega)}, \qquad (3)$$

where $S_x(\omega)$, $S_{yx}(j\omega)$ are the spectral and mutual spectral density of the input and output signals.

Because of the large volume of computations, it is frequently difficult to use these methods. Furthermore, due to the errors in determining the correlation functions iteration methods can lead to divergent iterative processes. The application of the frequency method is connected with the necessity of calculating the spectral density functions, which is not always convenient.

Work [2] introduced a convenient method of finding the transfer function of a system that does not require calculating the cross-correlation function of the signals or the corresponding mutual spectral density. It is based on the concept of the noise weight function $K_\delta(\tau)$, which is the correlation function of the reaction of the system to an input signal and the form of a unit of white noise. By using the relationships that connect the bilateral Laplace transform of the noise weight function $K^*_\delta(p)$ and the correlation functions of the input and the output signals $K^*_{in}(p)$ and $K^*_{out}(p)$ in the form

$$K^*_{out}(p) = K^*_\delta(p) K^*_{in}(p), \tag{4}$$

and also the connection of the bilateral Laplace transform of the noise weight function and the transfer function of the system W(p), which has the form

$$K^*_\delta(p) = W(p)\,W(-p),$$

from the known correlation functions of the signals we can determine the transfer function of the system W(p) and, consequently, the pulsed transfer function g(t).

The method is brought to a specific result for the case when the correlation functions of the signals of the system have the form

$$K_{in}(\tau) = A_0 e^{-\alpha_1|\tau|}(\cos\beta_1|\tau| + A\sin\beta_1|\tau|), \tag{5a}$$

$$K_{out}(\tau) = B_0 e^{-\alpha_2|\tau|}(\cos\beta_2|\tau| + B\sin\beta_2|\tau|). \tag{5b}$$

In actuality, simultaneous fulfillment of equalities (5a) and (5b) is encountered rather rarely. If the correlation functions of the signals are presented by different dependences, there can be ambiguity in determining the transfer function. As A. N. Sklyarevich has shown [3], this ambiguity can be removed by including additional information about the system or about its output signals, which

significantly expands the possibilities of this method.

A. N. Sklyarevich proposed another method for finding the transfer function of the system W(s). It is based on representing the cross-correlation function of the two nonstationary processes $K_{yx}(t, t_1)$ of the input and output signals

$$K_{yx}(t,t_1)=\int_0^t g(t-u)K_x(|u-t_1|)\,du \tag{6}$$

when $t \leq t_1$, by an expression of the form

$$K_{yx}(t,t_1)=\int_0^t g(t-u)K_x(t_1-u)\,du, \tag{7}$$

and when $t \geq t_1$, by the expression

$$K_{yx}(t,t_1)=\int_0^{t_1} g(t-u)K_x(t_1-u)\,du+\int_{t_1}^{t} g(t-u)K_x(u-t_1)\,du. \tag{8}$$

For stationary processes equations (7) and (8) can be rewritten in another form (considering that $t - t_1 = \tau$):

$$K_{yx}(\tau)=\int_0^\infty g(u)K_x(\tau+u)\,du \quad (\tau\leqslant 0) \tag{9}$$

and

$$K_{yx}(\tau)=\int_0^\infty g(\tau+u)K_x(u)\,du+\int_0^\tau g(\tau-u)K_x(u)\,du \quad (\tau\geqslant 0). \tag{10}$$

When the transfer function of the system and the univariate Laplace transform of the correlation function of the input signal are proper fractions with simple poles α_i and β_j in the left half-plane of the argument s and when these poles do not coincide (correlation resonance is not observed):

$$K_x(\tau)=\sum_{i=1}^{n} A_i e^{\alpha_i|\tau|}, \tag{11a}$$

$$g(t)=\sum_j B_j e^{\beta_j t}\,(Re\,\alpha_i<0;\ Re\,\beta_j<0), \tag{11b}$$

the expression for $K_{yx}(\tau)$ when $\tau \geq 0$ can be written in the form

$$K_{yx}(\tau)=\sum_{i=1}^{n} A_i W(\alpha_i) e^{\alpha_i|\tau|}+\sum_{j} B_j [F_x(\beta_j)+F_x(-\beta_j)] e^{\beta_j|\tau|}, \tag{12}$$

where $F_x(s)$ is the univariate Laplace transform of the correlation function $K_x(\tau)$.

When the cross-correlation function can be presented in the form

$$K_{yx}(\tau)=\sum_{i=1}^{n} A_i^{*} e^{\alpha_i \tau}+\sum_{j} B_j^{*} e^{\beta_j \tau} \quad (\tau>0), \tag{13}$$

after comparing the right parts of formulas (12) and (13) we can determine the expansion coefficients of the pulsed transfer function

$$B_j=\frac{B_j^{*}}{F_x(\beta_j)+F_x(-\beta_j)} \tag{14}$$

and, consequently, we can find g(t) in accordance with formula (11b).

Let us show how this method can be expanded for the case when the poles of the transforms of the input signal correlation function and the transfer function of the system have an order higher than one.

Let us examine this problem: from the known input signal correlation function $K_x(\tau)$ and the known pulsed transfer function of the system g(t), taking the conditions of the physical possibility of the system into account [$g(t) \equiv 0$ when $t < 0$], let us calculate the cross-correlation function of the input and output of the system $K_{yx}(\tau)$.

The Laplace transform of the function $K_x(\tau)$ and the transfer function of the system W(s) can be presented in the form

$$F_x(s)=\sum_{l=1}^{L} \sum_{v=1}^{V} \frac{A_{lv}}{(s-\alpha_l)^{v}}, \tag{15}$$

$$W(s)=\sum_{k=1}^{K} \sum_{r=1}^{R} \frac{B_{kr}}{(s-\beta_k)^{r}}. \tag{16}$$

In accordance with the examined case we assume that the poles of the functions $F_x(s)$ and $W(s)$ do not coincide:

$$\beta_k \neq \alpha_l.$$

The correlation function $K_x(\tau)$ and the pulsed transfer function of the system $g(t)$ are connected with transforms (15) and (16) by the inversion formula

$$Z(t) = \frac{1}{2\pi j} \int_{a-j\infty}^{a+j\infty} z(s) e^{st} ds,$$

where a is the abscissa of absolute convergence. Applying the inversion formula to expressions (15) and (16), we obtain

$$K_x(u) = \sum_{l=1}^{L} \sum_{v=1}^{V} A_{lv} \frac{u^{v-1} e^{\alpha_l u}}{(v-1)!}, \tag{17}$$

$$g(\tau - u) = \sum_{k=1}^{K} \sum_{r=1}^{R} B_{kr} \frac{(\tau - u)^{r-1} e^{\beta_k (\tau - u)}}{(r-1)!}. \tag{18}$$

Since

$$(\tau - u)^{v-1} = \sum_{j=0}^{v-1} \frac{(-1)^j (v-1)!}{(v-1-j)!\, j!} \tau^{v-1-j} u^j \tag{19}$$

and

$$\int x^n e^{ax} dx = e^{ax} \sum_{i=0}^{n} \frac{(-1)^i x^{n-i} n!}{a^{i+1} (n-i)!}, \tag{20}$$

taking formulas (13), (17) and (18) into account, we obtain an expression for the cross-correlation function $K_{yx}(\tau)$:

$$K_{yx}(\tau) = \sum_{k=1}^{K} \sum_{r=1}^{R} B_{kr} \sum_{j=0}^{r-1} \frac{\tau^{r-1-j} (-1)^j}{(r-1-j)!\, j!} \sum_{l=1}^{L} e^{\alpha_l \tau} \sum_{v=1}^{V} \frac{A_{lv}}{(v-1)!} \times$$
$$\times \sum_{i=0}^{v-1+j} \frac{(-1)^i \tau^{v-1+j-i} (v-1+j)!}{(v-1+j-i)!\, (\alpha_l - \beta_k)^{i+1}} - \sum_{k=1}^{K} \sum_{r=1}^{R} B_{kr} e^{\beta_k \tau} \sum_{l=1}^{L} \sum_{v=1}^{V} \frac{A_{lv}}{(v-1)!} \times$$
$$\times \sum_{j=0}^{r-1} \frac{\tau^{r-1-j} (-1)^{v-1+j} (v-1+j)!}{(r-1-j)!\, j!} \left[\frac{1}{(\alpha_l + \beta_k)^{v+j}} + \right.$$
$$\left. + \frac{(-1)^j}{(\alpha_l - \beta_k)^{v+j}} \right] \quad (\tau \geqslant 0). \tag{21}$$

From expression (21) it is evident that the cross-correlation function is a combination of sums that contain exponential terms corresponding to both the input signal correlation function $K_x(\tau)$ (17) and the pulsed transfer function g(t) (18). Consequently, to solve the reverse problem, determining the pulsed transfer function from experimentally calculated correlation functions, it is expedient to present the cross-correlation function in the form

$$K_{yx}(\tau)=\sum_{l=1}^{L}\sum_{d=1}^{D} M_{ld}\tau^{d-1}e^{\alpha_l\tau}+\sum_{k=1}^{K}\sum_{n=1}^{N} M_{kn}\tau^{n-1}e^{\beta_k\tau} \quad (\tau\geqslant 0). \tag{22}$$

Calculating the coefficients at each power τ of the exponential components $e^{\beta_k\tau}$ of the sought pulsed transfer function, equating them to the coefficients M_{kn} of the experimentally calculated cross-correlation function (22) and introducing the designation

$$-\sum_{l=1}^{L}\sum_{v=1}^{V} A_{lv}(-1)^{v-1+j}C_{v-1+j}^{v-1}\left[\frac{1}{(\alpha_l+\beta_k)^{v+j}}+\frac{(-1)^j}{(\alpha_l-\beta_k)^{v+j}}\right]=a_{k,j+1}, \tag{23}$$

we arrive at the following system of equations:

$$\left.\begin{aligned}
&M_{kN}(R-1)!=B_{kR}a_{k1};\\
&M_{kN-1}(R-2)!=B_{kR}a_{k2}+B_{kR-1}a_{k1};\\
&M_{kN-2}(R-3)!=B_{kR}a_{k3}+B_{kR-1}a_{k2}+B_{kR-2}a_{k1};\\
&\dots\dots\dots\dots\dots\dots\dots\dots\\
&\dots\dots\dots\dots\dots\\
&M_{k1}0!=B_{kR}a_{kR}+B_{kR-1}a_{kR-1}+\dots+B_{k2}a_{k2}+B_{k1}a_{k1},
\end{aligned}\right\} \tag{24}$$

where R and N are the maximal orders of the pole β_k in expressions (16) and (22). With system (24) we can easily determine the unknown coefficients of the pulsed transfer function

$$B_{kr}=\frac{\Delta_r}{\Delta}, \tag{25}$$

where Δ is the determinant of system (24); Δ_r is the minor obtained by replacing the r-th column with the left part of system (24).

From system (24) we can derive a recurrent formula for determining the unknown coefficients

$$B_{kr} = \frac{M_{km}(r-1)! - \sum_{l=1}^{R-r} B_{kr+l}\, a_{kl+1}}{a_{k1}}. \qquad (26)$$

In a particular case we can show that expression (14) follows from formula (26) when r = v = 1 (the case of simple poles).

Thus, we can propose the following procedure for determining the transfer function:

- using the recordings of the signal realizations on the input and output of the system, by any of the known methods calculate the correlation function $K_x(\tau)$ and the cross-correlation function $K_{yx}(\tau)$ ($\tau \geq 0$);
- approximate the graphs of the correlation functions by analytical expressions similar to (17) and (22);
- number the powers of the terms of the expansion of the correlation function $K_{yx}(\tau)$;
- calculate the values of the coefficients a_{kj+1} from formula (23);
- determine the coefficients B_{kr} from formula (25) or formula (26);
- present the transfer function of the system in the form of (16) or, when necessary, transform it.

This method of determining the transfer function is an expansion of the method proposed in work [3] for the case when the poles of the transfer function of the system W(s) and the transforms of the input signal correlation function $F_x(s)$ have an order higher than one. This significantly increases the possibilities of the given method by creating prerequisites for alternating the calculation processes in determining the characteristics of the system during its normal exploitation.

Examples

1. The input of the object is fed a random process with the correlation function

$$K_x(\tau)=e^{-|\tau|}.$$

The behavior of the cross-correlation function in the region of positive values of the argument is approximated well enough by the dependence

$$K_{yx}(\tau)=\frac{5}{4}e^{-\tau}-\frac{2}{3}e^{-2\tau}-e^{-3\tau}(0,25\tau+0.1875).$$

Determine the transfer function of the object.

The coefficients M_{kn} (22) have the following values:

$$M_{11}=-\frac{2}{3};\quad M_{21}=-0,1875;\quad M_{22}=-0,25.$$

From formula (23) find the values of a_{kj+1}

$$a_{11}=-(-1)^0\left[\frac{1}{-1-2}+\frac{1}{-1+2}\right]=-\frac{2}{3};$$

$$a_{21}=-(-1)^0\left[\frac{1}{-1-3}+\frac{1}{-1+3}\right]=-\frac{1}{4};$$

$$a_{22}=-(-1)^1\left[\frac{1}{(-1-3)^2}-\frac{1}{(-1+3)^2}\right]=-\frac{3}{16}.$$

Using formula (26), find

$$B_{11}=1;\quad B_{21}=0;\quad B_{22}=1.$$

Thus, the transfer function has the form

$$W(s)=\frac{1}{s+2}+\frac{1}{(s+3)^2}.$$

2. The input of the system is fed a random process with the correlation function

$$K_x(\tau)=e^{-3|\tau|}.$$

The cross-correlation function in the region $\tau \geq 0$ is approximated by the dependence

$$K_{yx}(\tau)=e^{-\tau}\left[\left(\frac{108}{85}\tau+\frac{2124}{3757}\right)\cos\tau+\left(\frac{24}{85}\tau+\frac{948}{3757}\right)\sin\tau\right]+$$
$$+\frac{12}{5}e^{-2\tau}+e^{-3\tau}\left(\tau\frac{8}{5}-\frac{41}{25}\right).$$

Since

$$\sin\tau=\frac{e^{\tau j}-e^{-\tau j}}{2j},$$
$$\cos\tau=\frac{e^{\tau j}+e^{-\tau j}}{2},$$

we bring $K_{yx}(\tau)$ to the form

$$K_{yx}(\tau)=e^{-\tau}\left\{\tau e^{\tau j}\left(\frac{1}{4-j}+\frac{1}{2+j}\right)+e^{\tau j}\left[\frac{1}{(4-j)^2}+\frac{1}{(2+j)^2}\right]+\right.$$
$$\left.+\tau e^{-\tau j}\left(\frac{1}{4+j}+\frac{1}{2-j}\right)+e^{-\tau j}\left[\frac{1}{(4+j)^2}+\frac{1}{(2-j)^2}\right]\right\}+$$
$$+\frac{12}{5}e^{-2\tau}+e^{-3\tau}\left(\tau\frac{8}{5}-\frac{41}{25}\right).$$

From this expression it is evident that

$$M_{11}=\frac{1}{(4-j)^2}+\frac{1}{(2+j)^2};\quad M_{12}=\frac{1}{4-j}+\frac{1}{2+j};$$
$$M_{21}=\frac{1}{(4+j)^2}+\frac{1}{(2-j)^2};\quad M_{22}=\frac{1}{4+j}+\frac{1}{2-j};\quad M_{31}=\frac{12}{5}.$$

Determine the coefficients a_{kj+1} (23)

$$a_{11}=\frac{1}{4-j}+\frac{1}{2+j};\quad a_{12}=\frac{1}{(4-j)^2}-\frac{1}{(2+j)^2};$$
$$a_{21}=\frac{1}{4+j}+\frac{1}{2-j};\quad a_{22}=\frac{1}{(4+j)^2}-\frac{1}{(2-j)^2};$$
$$a_{31}=\frac{6}{5}.$$

From formula (26) find the unknown coefficients

$$B_{11}=0;\quad B_{12}=1;\quad B_{21}=0;\quad B_{22}=1;\quad B_{31}=2;\quad B_{32}=0.$$

The transfer function has the form

$$W(s)=\frac{1}{(s+1-j)^2}+\frac{1}{(s+1+j)^2}+\frac{2}{s+2}=$$
$$=2\frac{s^2+2}{(s^2+2s+2)^2}+\frac{2}{s+2}.$$

Conclusions

1. The introduced formulas allow us to find the transfer function of the system with sufficient simplicity for the case when the poles of the transfer function and the transforms of the input signal correlation function have an order higher than one.

2. The obtained results can be used to develop special-purpose computers for automating the process of calculating the transfer function.

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